HOMOTHERMIC MOTION OF A GAS NEAR A DENSE MEDIUM

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Several studies ([1-3], for example) have examined problems which describe the motion of a gas with an energy release near the boundary between two media of different densities. The effect of the radiation of the medium on the laws of motion of the gas becomes important when the rate of energy release is high [4, 5].

Here, we use a homothermic model to solve a two-dimensional similarity problem on the propagation of an expansion shock from the boundary of a medium with a cavity.

Let energy E_0 in the form of radiation be instantaneously released per unit surface of the interface in a dense medium near a boundary with a cavity at $t = 0$. At $t > 0$, a radiative expansion shock $x = -x_1(t)$ propagates from the interface ($x = 0$) into the depth of the medium (into the region $x < 0$). Vaporization of the medium takes place on the shock front. The vaporized substance expands into the cavity and fills the region $x > 0$.

The mechanism of displacement of the interface between the dense medium and the vapor is as follows. A thin layer on the boundary with the cavity is instantaneously vaporized as a result of the high density of the released energy. Due to intensive heat exchange between the particles at high temperature (the mean free path of the radiation $\lambda_R \sim T^m \rho^{-n}$, $m, n > 0$), temperature is equalized throughout the disturbed region. The pressure gradient which arises leads to movement of the vaporized substance into the cavity, thereby helping to reduce density. This means that the mean free path of the radiation increases, and the next thin layer of the substance is heated and vaporized. Thus, as a result of radiant heat transfer and the motion of vapor toward the free surface, the dense medium is supplied with the energy needed for its vaporization.

With allowance for the large values of thermal conductivity, the process is assumed to be homothermic. The vaporized substance is modeled by an ideal gas. The dense medium is considered to be undeformable. We ignore losses of released energy in the vaporization of the medium and in the form of radiation from the free surface.

The system of equations which describes the unidimensional motion being examined has the form

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x},\ \frac{\partial T}{\partial x} = 0,\ \frac{\partial \rho}{\partial t} = -v\frac{\partial \rho}{\partial x} - \rho\frac{\partial v}{\partial x}.$$

Using the equation of state of an ideal gas $p = \rho RT/\mu$ and excluding pressure, we obtain

$$\begin{gathered}\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + a^2\frac{\partial}{\partial x}(\ln\rho) = 0,\\ \frac{\partial}{\partial t}(\ln\rho) + v\frac{\partial}{\partial x}(\ln\rho) + \frac{\partial v}{\partial x} = 0,\end{gathered} \tag{1}$$

where $a = \sqrt{RT/\mu}$ is the isothermal speed of sound. The laws of conservation of momentum and mass of the gas at the shock $x = -x_1$ lead to the equations

$$\rho_0 D = \rho_1(D + v_1),\ \rho_0 D^2 + \rho_0 a^2 = \rho_1(D + v_1)^2 + \rho_1 a^2. \tag{2}$$

Here, $D = dx_1/dt$ is the velocity of the shock; the indices 1 and 0 denote values behind and ahead of the shock, respectively.

With allowance for the assumptions already made, the energy of the moving gas is conserved, while the mass of the gas can be expressed through the parameters ρ_0 and x_1:

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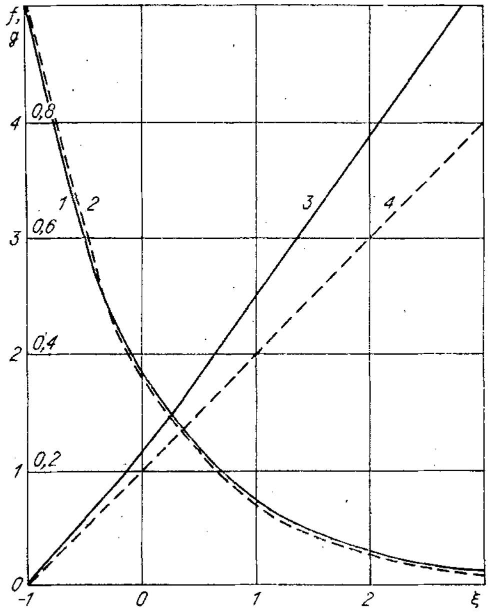

Fig. 1

$$\int_{-x_1}^{x_2} \rho\left(\frac{v^2}{2}+\frac{a^2}{\gamma-1}\right)dx = E_0, \quad \int_{-x_1}^{x_2} \rho dx = \rho_0 x_1 \tag{3}$$

(x_2 is the coordinate of the free surface). The motion of the gas described by Eqs. (1)-(3) is self-similar. We introduce the similarity variable

$$\xi = \beta x/x_1, \tag{4}$$

where $x_1 = \xi_0(E_0/\rho_0)^{1/3}t^{2/3}$; ξ_0 and β are constants subject to determination.

To find the velocity, density, and temperature of the gas, we can use the formulas

$$v = Df/\beta, \ \rho = \rho_0 g, \ T = D^2\mu/\beta^2 R. \tag{5}$$

Inserting (4) and (5) into (1), we obtain a system of ordinary differential equations

$$\frac{df}{d\xi} = \frac{(\xi - f)f}{2(1-(\xi-f)^2)}, \quad \frac{d}{d\xi}(\ln g) = (\xi - f)\frac{df}{d\xi} + \frac{f}{2}. \tag{6}$$

Changing over to dimensionless variables in (2), we obtain relations for the functions $f(\xi)$ and $g(\xi)$ on the shock at $\xi_1 = -\beta$:

$$f_1 = 1/\beta - \beta, \ g_1 = \beta^2. \tag{7}$$

The laws of conservation of the energy and mass of the gas (3) appear as follows in dimensionless form

$$\frac{\xi_0^3}{\beta^3}\int_{\xi_1}^{\xi_2} g\left(\frac{f^2}{2}+\frac{1}{\gamma-1}\right)d\xi = 1, \quad \int_{\xi_1}^{\xi_2} g d\xi = \xi_1. \tag{8}$$

Numerical solution of system (6) makes it possible to find the dimensionless velocity $f(\xi)$ and density $g(\xi)$ satisfying conditions (7) and (8). The position of the shock ξ_1 is found as the point of intersection of the integral curve of Eq. (6) with the curve of $f = (\xi^2 - 1)/\xi$. The values of ξ_0 and ξ_2 are determined from Eqs. (8).

Figure 1 shows calculated profiles of the dimensionless velocity and density (lines 3, 1). Shown for comparison are results of the solution of a similar problem for the isothermal case (lines 4, 2).

At T = const, the problem of the laws of propagation of the shock $x = -x_1 = -$ at has the analytic solution

$$\rho = \rho_0 e^{-1-x/at},\ v = a(1 + x/at),\ -x_1 \leqslant x < \infty,\ t > 0.$$

It is evident from the above results that in both the similarity solution and in the isothermal case, the velocity of the gas boundary in the cavity is infinite ($\xi_2 = \infty$). However, the total energy remains finite, since at $\xi \to \infty$ the density decreases more rapidly ($g \sim e^{-\xi}$) than the square of velocity increases ($f^2 \sim \xi^2$). The velocity of the shock in both cases is equal to the speed of sound ($\beta = 1$), $\xi_0 = 0.448$ at $\gamma = 1.11$.

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GROUP-INVARIANT SOLUTIONS AND INTERRELATION OF THE PARAMETERS OF THE MOISTURE-TRANSPORT EQUATION

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Study of the moisture-transport processes (during incomplete saturation of a porous medium by water) is a complex and urgent problem. Its component part is the reliable and rapid experimental determination of the parameters of saturated—unsaturated soil. Investigation of the exact solutions of the moisture-transport equation by the method of Lie groups [1] is used for these purposes. A general regularity of the exponential time dependence of the fluid mass flow rate [2], which had been encountered earlier [3], is detected in experiments on dehydration of a soil specimen. Its elucidation in the presence of a strong nonlinearity in the equations is given within the framework of group-invariant solutions. The conditions for expansion of the group result in interactions of the moisture-transport factor, the fundamental hydrophysical dependence, and the moisture, which can turn out to be useful for modeling the moisture transport processes.

1. Formulation of the Problem

The one-dimensional horizontal motion of water in an unsaturated porous medium is described by the moisture-transport equation [4]

$$\theta'_t = [K(p)\,p'_x]'_x, \tag{1.1}$$

where p is the pressure in water column units ($p < 0$ for incomplete saturation); K(p), moisture-transport factor; θ, volume humidity; t, time; and x, longitudinal coordinate.

We introduce a new function which we call the generalized head

$$F(p) = \int K(p)\,dp. \tag{1.2}$$

Then (1.1) takes the form

$$\theta'_t = F''_{x^2}. \tag{1.3}$$

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